

TC2 - S.Q.

TC2  **TECNIDEA CIDUE**
S.r.l.

...ideas in motion...

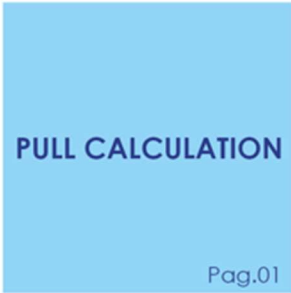
n°27
March 2024

NEWSPAPER
Technical News



...ideas in motion...

TOPICS TREATED: Belt transmission



Drafting by Dott. Giorgio Canova

Franco Canova: Ideas, Creations, Inventions

BELT TRANSMISSIONS: Pull calculation and tensioner definition

Belt transmissions are usually constituted of a closed ring belt and two pulleys, a motor and a driven one. There are many types of belts on the market, but the most utilized are: flat belts, round belts and Poly-V belts but mostly V-belts executed in the different sections: Z-A-B-C-D-E etc. For toothed belts consult the dedicated chapters for they are positive transmissions. Even if there various types of belts exist all of them comply to a main functioning principle that determines the design base rules, calculations and execution (img. 1)

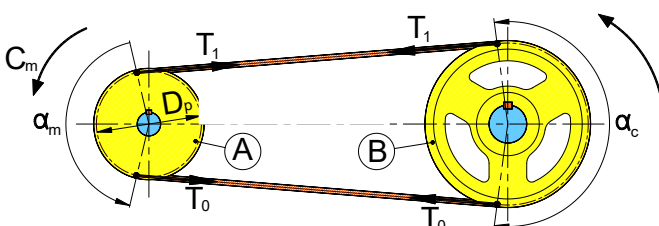


fig. 1

- A = Motor Pulley
- B = Driven pulley
- D_p = Motor pulley pitch diameter in mm
- T_1 = tension on tensioned arm in Newton
- T_0 = Tension on driven arm in Newton
- C_m = Maximum momentum on motor shaft in Newton
- M_t = Momentum to be transmitted operatively in Newton
- α_m = Winding angle Motor pulley
- α_c = Winding angle driven pulley



- Ideal application conditions are given by:
 - Correct development of the tense belt
 - Full contact of the belt on the pulleys in correspondence to the winding angles α_m and α_c

With these premises it is clear that the ideal condition to grant the correct functioning (Img.2) must satisfy the equation:

$$F \leq T_1 \tag{eq. a}$$

And that

$$T_1 = T_0 \tag{eq. b}$$

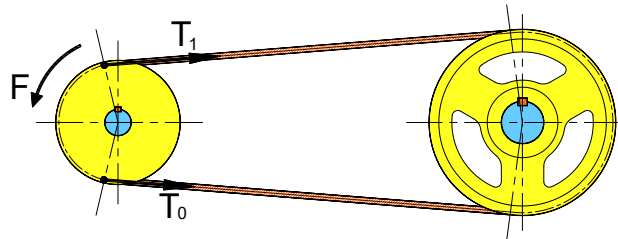


fig. 2

- Unfortunately real working conditions present criticalities, first it's not always granted the transmission ratio because of inevitable errors and inconveniences such as:
 - Incorrect belt development
 - Low winding angles α_m (motor pulley) and α_c (driven pulley)
 - Low friction coefficient " η " for direct or indirect causes
 - Vibrations or intermittent functioning
 - insufficient belt tensioning

Each of the above mentioned anomalies can be cause of slippage of the belt on the pulleys and functioning errors such as speed variation and reduction of the transmitted torque. To eliminate them an automatic belt tensioner is needed to recover elongations, absorb vibrations, increase winding angles but most of all grant the tension of the belt; Its use creates a node "n" in the contact point between the tensioner roller and the belt and it is in this node "n" position where the different forces in play manifest and produce the resulting T_0 and T_1 that determine the minimal equilibrium condition needed for the production of movement (Img. 3).

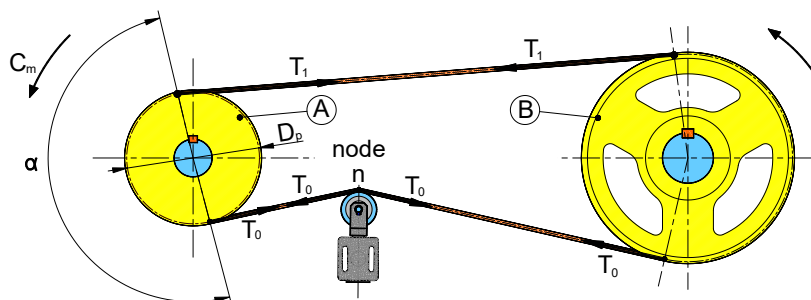
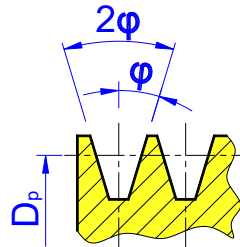


fig. 3



The belt tensioner has to be chosen based on the pull of the two belt arms, “ T_1 ” and T_0 ”; and their value is calculated by writing the equilibrium and limit equations on the transmission, those are “equilibrium equation at the rotation of the driven pulley” (equation “c”), together with “slippage limit condition” (eq. “d”); because the winding angle on the motor pulley α_m is lower than α_c and must be at least 180° corresponding to πrad ; so the system that allows for the pulls calculation T_1 and T_0 is:

$$\begin{cases} (T_1 - T_0) \times \frac{D_p}{2} \times \frac{1}{1000} = C_m & \text{(eq.c)} \\ T_1 = T_0 e^{\eta\alpha} & \text{(eq.d)} \end{cases}$$



- e = Nepero's number, constant equal to 2,72
- η = Friction coefficient between belt and pulley (in the case of V-belts it has to be divided by $\sin(\varphi)$, where φ is the half opening angle of the groove measured in rad).
- M_t = Momentum to transmit working in Nm
- C_m = Maximum momentum on motor arm Nm
- f_s = Service factor from 2 to 5

fig. 4

As it's known, every machines has critical points, such as their start or work peaks for which it is necessary to multiply the torque to transmit “ M_t ” (see next equation “h”) for the service factor “ f_s ” to obtain the maximum working torque

“ C_m ”: $C_m = f_s \times M_t$ (see next eq. “d”)

$f_s = 2 \div 5$ (eq. “e”)

The belt tensioner has to be installed on the slow arm, the closest possible to the motor pulley and has to develop enough force to equilibrate the result of the sum of the two components of the transmission of the slow arm. For the correct functioning of the tensioning element you need to respect the indications on page 15 and 16 for axial tensioners and on page 19 and 20 for rotational tensioners. For the correct functioning of the belt tensioner it is essential for it to be positioned in a way that the entry and exit angles “ δ ” of the node are equal (img.5). Image 5 shows the correct assembly of the tensioner on the slow arm of the transmission; The configuration is such to form an exit angle of the belt from the motor pulley of γ degrees in respect to the vertical, while on the driven one of β degrees in respect to the vertical; this makes it so that the working angles “ δ ” measure:

$$\delta = \frac{1}{1000} \frac{(180^\circ - \gamma - \beta)}{2} \quad \text{(eq. “f”)}$$

The above described conditions are the minimal equilibrium conditions for the functioning of the belt; In the following we present a calculation example.

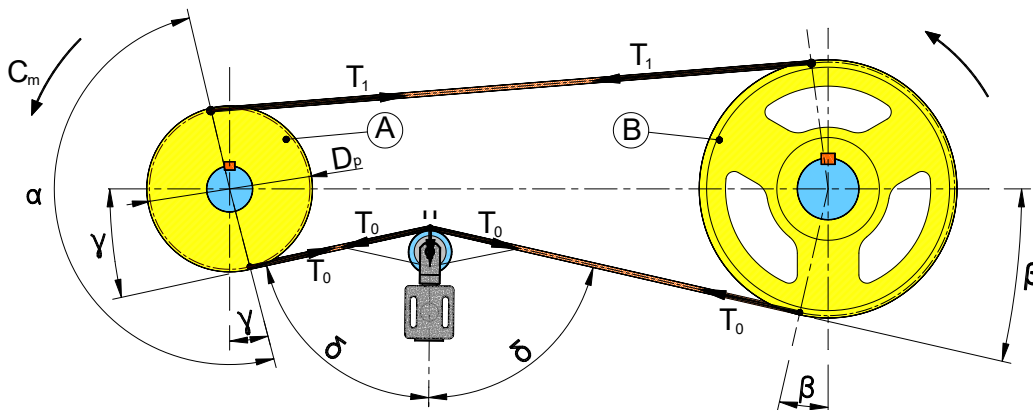


fig. 5



Tensioner choice on belt transmission

- Calculation example:

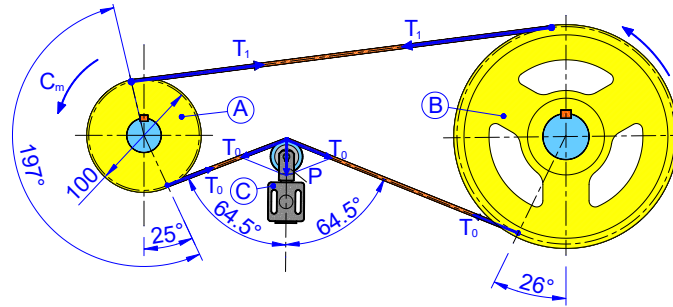


fig. 6

- Motor specifics: $N=3 \text{ Cv}$

$$n=940 \text{ rpm}$$

Transform the precedent values with the SI measurement units:

$$N=3 \times 735 = 2205 \text{ W}$$

$$\omega = 940 \times \pi / 30 = 98,4 \text{ rad/s}$$

$$N = M_t \times \omega \rightarrow M_t = N / \omega = 22,4 \text{ Nm}$$

It is supposed that $f_s = 2,5$

$$C_m = 2,5 \times M_t = 56 \text{ Nm}$$

Motor pulley pitch diameter $D_p = 100 \text{ mm}$

$$\begin{cases} (T_1 - T_0) \times 0,05 = 56 \rightarrow (T_1 - T_0) = 1120 & \text{(esp. c)} \\ T_1 = T_0 e^{\eta \alpha} & \text{(esp. d)} \end{cases}$$

- Winding angle $\alpha = 197^\circ \times \pi / 180^\circ = 3,44 \text{ rad}$
- Friction coefficient between belt and pulley $\eta = 0,2$
- V-belt with semi opening angle $\varphi = 17^\circ \rightarrow \sin(\varphi) = 0,29$
- V-belt $\rightarrow \eta' = 0,2 / \sin(\varphi) = 0,2 / 0,29 = 0,69$
- Nepero's number $e = 2,72$

$$\begin{cases} T_1 = T_0 \times e^{0,69 \times 3,44} = T_0 \times 10,74 & \text{(eq. d)} \\ (10,74 T_0 - T_0) = 1120 & \text{(eq. c)} \end{cases}$$

$$\rightarrow T_0 = 115 \text{ N}$$

$$\rightarrow T_1 = 1120 + 115 = 1235 \text{ N}$$

$$\rightarrow P = 2 \times 115 \times \cos(64,5^\circ) = 99 \text{ N}$$



Given the force value F , we can choose the tensioner, this must develop a maximum push 30% superior, so it can be installed on the machine in a range in which the required force is at most the 2/3 of its push to obtain a pressure that grants the motion transmission in a consolidate equilibrium position.

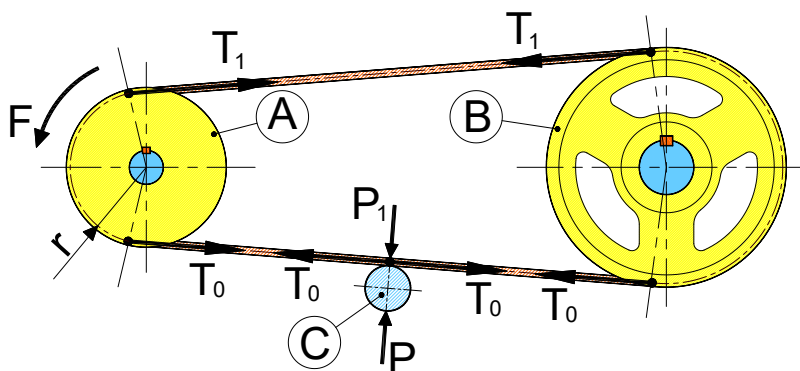
Everything that's been explained till now only concerns belt transmissions working in ideal conditions and considering the highest level of functioning, thus in "equilibrium" condition ; but reality is much different and working conditions keep on changing according to the external factors that intervene, let's quote the most common ones:

- Starts
- Stops
- Load peaks
- Speed variations
- Overcharge
- Errors in the belt development
- Environmental conditions
- Working temperature variation
- Work cycles
- Pulleys wear
- Belts wear
- Conditions and wearing agents
- Specific criticalities

Each of these incidences modify the geometry and the state of the transmission, so to increase the machines reliability You need to intervene beforehand in a way that prevents eventual criticalities. Representing the typical belt transmission, *Img.7*, we can see that motion is granted in the moment in which the transmission equations are satisfied; in which the force "N" (eq. "g") and the transmitted torque "Mt" (eq. "h") depend on the rpm "n" so:

$$N = \frac{Mt \times n}{716,2} \quad (\text{esp. "g"})$$

$$Mt = \frac{N \times 716,2}{n} \quad (\text{esp. "h"})$$



- A = Motor pulley
- B = Driven pulley
- C = Belt tensioner
- r = radius = b (arm)
- T₁ = tension on tensioned arm
- T₀ = tension on driven arm
- F = driving force
- P = force produced by the belt tensioner
- P₁ = reaction

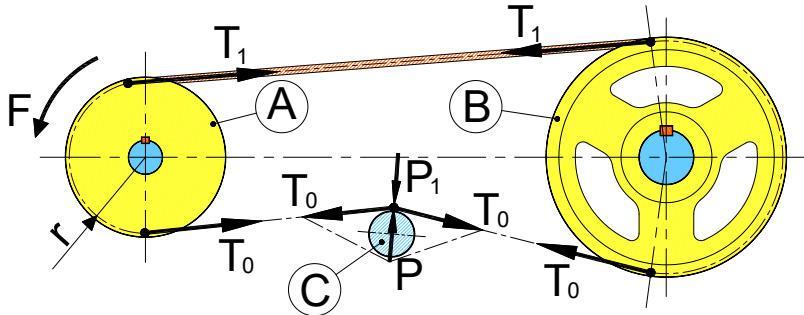
Img. 7



The torque Mt is also obtained by: $Mt = F \times b$ (eq. "i")

For this reason $F = \frac{Mt}{b}$ (equation "l")

From these equations we deduce that the equilibrium condition is represented by $F \leq T_1$ because $T_1 = T_0$ $F \leq T_0$ (eq. "m")

The belt tensioner application creates a node "n" in which the forces in play emerge; which modifies the geometry of the transmission; For the effect of the force "P" produced by the belt tensioner the two belt's arms move the node "n" higher till it reaches the equilibrium state P_1 that is the reaction to P and creating the angle " β " between the two belt's arms; the system's equilibrium forces have to manifest on these vectors 

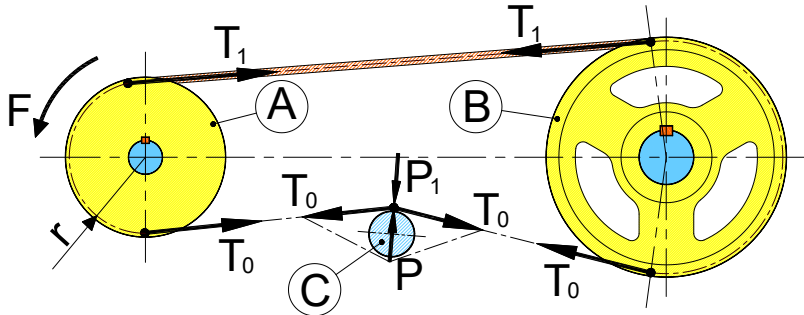
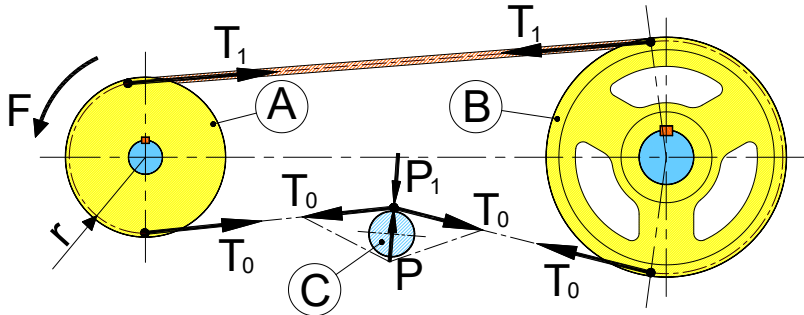
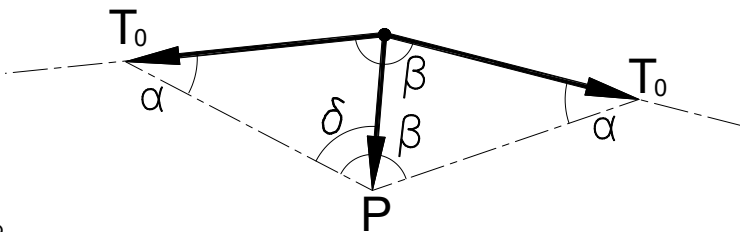
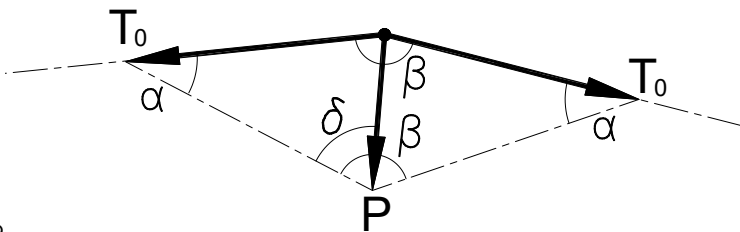


fig. 8

To determine the force of the belt tensioner to use you have to construct the parallelogram of the system's forces having as angles β, α and ϱ emerged from the equilibrium status for the effect of the force P . Because of the known force "F" for the equilibrium status at the motion we also determine T_1 and $T_0 \geq F$ so we can construct the graph  and solve the following equation:

$$\varrho = \frac{\beta}{2} \quad P = \frac{T \times \text{sen } \alpha}{\text{sen } \varrho}$$



Img. 9

In short, a belt transmission is granted when the belt is always adequately in pull and this is maintained by the breakdown of the force of the belt tensioner that based on the β produces the corresponding forces T at the minimum pull to grant the force F.

Edited by: Franco Canova